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AN ENHANCED SELECTION OPERATOR FOR MULTI-OBJECTIVE OPTIMIZATION DIFFERENTIAL EVOLUTION ALGORITHM

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ABSTRACT: *Evolutionary algorithms have been proven to handle multi objective problems, and one of such finest algorithms is the Differential Evolution Algorithm. In the last five years, Differential Evolution (DE) has been used to solve multi objective optimization problems (MOOPs). Several extensions of DE for multi-objective optimization have already been proposed. Older approaches convert a MOOP to a single-objective problem and use DE to solve the single objective problem, whereas more recent and advanced approaches mainly use the concept of Pareto-dominance. As the number of objectives increases to four or more it is difficult finding the dominated solution as a result there exist conflicts among the objectives. In this research work, a method of controlling the dominance area of solutions using the Generalized Differential Evolution 3 (GDE3) Algorithm is proposed. Controlling the dominance area means either the expansion or contraction of the dominance area solutions using a user-defined parameter S. Deb-Thiele-Laumanns-Zitzler (DTLZ) test problems were used to benchmark the performance of the proposed DE algorithm.*

KEYWORDS: Evolutionary Algorithm, MOOP, GDE, CDAS, CEDA

INTRODUCTION

Optimizing a problem involves finding the best possible values of the deciding factors of the problem. The fine-tuned values of these variables or factors either maximize or minimize the solution of the given problem. The problem may have either a single solution or multiple conflicting solutions. These are termed as Single-objective Optimization Problem (SOP) and Multi-objective Optimization Problem (MOP) respectively. Among many existing algorithms, Evolutionary Algorithms (EA) were found to be effective in finding near optimal solutions. Some of the evolutionary algorithms are Genetic Algorithms (GA), Evolution Strategies (ES), and Differential Evolution (DE).

Nowadays, most of the real time optimization problems are of multi-objective type. Multi-objective optimization problems (MOP) can be seen in many fields like science, engineering, economics, etc. A MOP consists of more than one conflicting objective functions. In MOP, single solution that can simultaneously optimize all objective functions does not exist, instead it will have set of solutions that are optimal, which is called Pareto front. There are two requirements of MOP; (i) to find out a solution that converges to the pareto optimal front (ii) to find solution that will maintain diversity in population.

A classical approach for multi-objective optimization is to convert a MOOP into a single objective form by predefining weighting factors for different objectives, expressing the relative importance of each objective apriori. The weights then define what kind of compromise solution is sought by a decision-maker. The decision-maker selects the final solution among the sets of non-dominated solutions that are equally good in the sense of Pareto-dominance

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(Miettinen and Makela, 2000). Other ways than weights also exist to express the preference of the decision maker apriori, e.g. e-constraint and goal programming methods. In a situation where the decision maker is not able to provide relative importance of objectives beforehand, a better approach is to find a set of solution candidates and pick a solution which provides a suitable compromise between the objectives. This can be viewed as a posteriori articulation of the preferences of the decision maker concerning the relative importance of each objective. Besides apriori and a posteriori approaches, no preference and interactive approaches also exist (Miettinen and Makela, 2000). The no-preference approaches provide a solution without any preference information and the interactive approaches involve the decision-maker interacting/guiding the solution process.

Review of other extensions of differenial evolution algorithms for multi-objectives optimization problems

Proposed by	**Year**	**Differential Evolution (DE) Extensions**
Abbass et al	2001	Pareto (frontier) DE algorithm
Lampinen	2002	First Version, Generalized Differential Evolution (GDE 1)
Madavan	2002	Pareto DE Approach (PDEA)
Zaharie	2003	Adaptive Pareto DE (APDE)
Parsopoulos et al.	2004	Vector Evaluated DE (VEDE)
Iorio and Li	2004	Non-dominated Sorting DE (NSDE
Robi£ and Filipi£	2005	DE for Multi-objective Optimization (DEMO)
Coello	2005	e-MyDE
Justesen and Ursem	2009	Cluster-Forming Differential Evolution (CFDE)
Montaño, Coello Coello, and Mezura-Montes	2010	Multi-Objective Differential Evolution with Local Dominance and Scalar Selection (MODE-LD+SS)
Zhong and Zhang	2011	Adaptive Multi-Objective Differential Evolution with Stochastic Coding Strategy (A-MODE SCS)
Ali, Siarry, And Pant	2012	Multi-Objective Differential Evolution Algorithm (MODEA)
Denysiuk et al	2013	Many-Objective DE with Mutation Restriction (M-ODEMR)

METHODOLOGY

Control Dominance Area of Solution (CDAS)

Controlling the dominance area of solutions controls the degree of expansion or contraction of the dominance area of solutions using a user-defined parameter *S*. By modifying the dominance area of solutions, it changes their dominance relation, inducing a ranking of solutions that is different to conventional dominance. The user defined variable could be either $S<0.5$ for expansion or $S>0.5$ for contraction which is Contrary to *e*-dominance and α-domination, which are relaxed forms of Pareto dominance where $S=0.5$. Some current research reveals that ranking by Pareto dominance on problems with an increased number of objectives might no longer be effective. It has been shown that the characteristics of multi-objective landscapes viewed in terms of non-dominated fronts can change drastically as the number of objectives increases, i.e. the number of fronts reduces substantially and become denser (more solutions

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per front) just by increasing the number of objectives. That is, most solutions are assigned the same rank of non-dominance and Pareto selection weakens.

Contraction and Expansion of Dominance Area (CEDA)

Normally, the dominance area is uniquely determined with a fitness vector $\boldsymbol{f}(\boldsymbol{x}) = (f1(\boldsymbol{x}), f2(\boldsymbol{x}), \cdots, fm(\boldsymbol{x}))$ in the objective space when a solution $\boldsymbol{x}$ is given. To contract and expand the dominance area of solutions, we modify fitness value for each objective function by changing the user defined parameter Si in the following equation.

$$f_i'(x) = \frac{r\,.\sin(\omega_i + S_i.\pi)}{\sin(S_i\,.\ \pi)}\ for\ all\ objectives\ 1, 2, \ldots, m$$

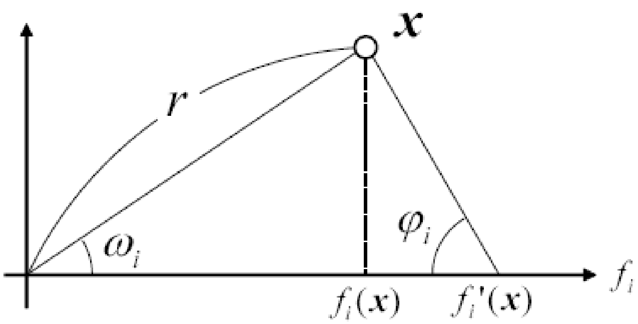

Figure 1: Transforming an objective function value

Where $\phi i = Si.\pi$. This equation is derived from the Sine theorem. We illustrate the fitness modification in Figure. 1, where r is the norm of $f(\boldsymbol{x})$, $fi(\boldsymbol{x})$ is the fitness value in the i-th objective, and ωi is the declination angle between $f(\boldsymbol{x})$ and $fi(\boldsymbol{x})$. In this example, the i-th fitness value $fi(\boldsymbol{x})$ is increased to $f'i(\boldsymbol{x}) > fi(\boldsymbol{x})$ by using $\phi i < \pi/2$ (Si <0.5). In case of $\phi i = \pi/2$ (Si = 0.5), $fi(\boldsymbol{x})$ does not change and $f'i\ (\boldsymbol{x}) = fi(\boldsymbol{x})$. Thus, this case is equivalent to the conventional dominance.

On the other hand, in case of $\phi i > \pi/2$ (Si >0.5), $fi(\boldsymbol{x})$ is decreased so $f'i\ (\boldsymbol{x}) < fi(\boldsymbol{x})$. Such fitness modification changes the dominance area of solutions. We show an example in Figure 2 (a)-(c), where three solutions ***a***, ***b*** and ***c*** are distributed in 2-dimensional objective space.

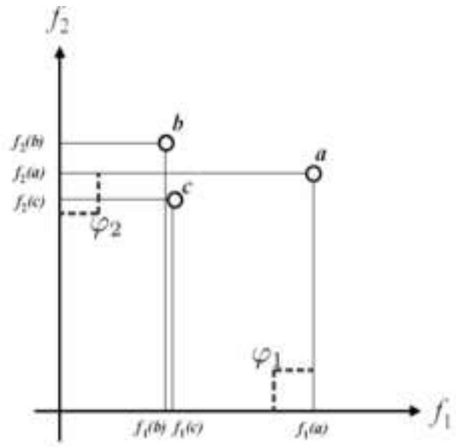

Figure 2(a) ***Si=0.5***

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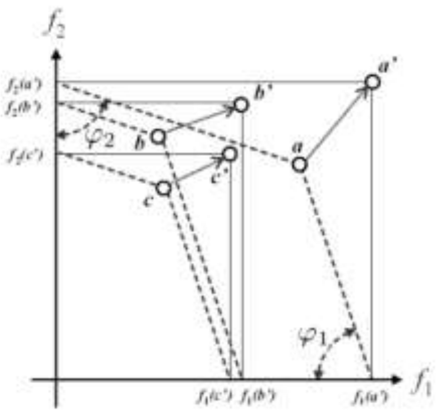

Figure 2(b) Si<0.5

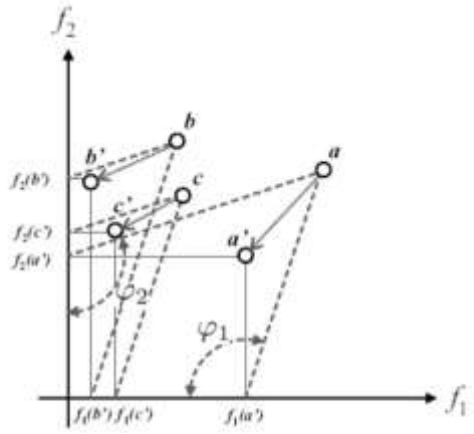

Figure 2(c) Si>0.5

In Figure 2 (a), ***a*** dominates ***c***, but ***a*** and ***b*** however, if we modify fitness values for each solution by using the equation, the location of each solution moves in the objective space, and consequently the dominance relationship among solutions changes. For example, if we use *S*1 = *S*2 <0.5 as shown in Figure 2 (b), the dominance area of solutions ***a’***, ***b’*** and ***c’*** is expanded from the original one of ***a***, ***b*** and ***c***. This causes that ***a’*** dominates ***b***’ and ***c’***, and ***b***’ dominates ***c’***. That is, expansion of dominance area by smaller *Si*(<0.5) works to produce a more fine grained ranking of solutions and would strengthen selection. On the other hand, if we use *S*1 = *S*2 >0.5 as shown in Figure 2 (c), the dominance area of solutions ***a’***, ***b’*** and ***c’*** is contracted from the original one of ***a***, ***b*** and ***c***. This causes that ***a’***, ***b’*** and ***c’*** do not dominate each other. That is, contracting the area of dominance by larger *Si* (>0.5) works to produce a coarser ranking of solutions and would weaken selection.

Proposed Multi-Objective Algorithm

This research suggests that for selection to be effective a more careful analysis of Pareto dominance relation is required when dealing with problems that have more than three objectives. In addition, for any number of objectives, the dominance relation should be appropriately revised according to the characteristics of the multi-objective landscape. In this research work, we proposed a method to control the dominance area of solutions in order to induce appropriate ranking of solutions for the problem at hand, enhance selection, and improve the performance of Multi-Objective Evolutionary Algorithm (MOEA) on combinatorial optimization problems. The proposed method can control the degree of expansion or contraction of the dominance area of solutions using a user-defined parameter *S*. Modifying the dominance area of solutions changes their dominance relation inducing a ranking of solutions that is different to conventional dominance. Contrary to *e*-dominance and α-domination, the proposed method can strengthen or weaken selection by expanding or contracting the area of dominance and conceptually can be considered as a generalization of Pareto dominance.

We also analyse the effects on solutions ranking caused by contracting and expanding the dominance area of solutions and its impact on the search performance of a multi-objective optimizer when the number of objectives increases, the size of the search space, and the complexity of the problems vary. The Generalized Differential Evolution 3 (GDE3) algorithm is chosen as the representative elitist algorithm that uses pareto dominance and compare its performance with GDE3 enhanced by the proposed method. The *m* = *{3, 6,* 8*}* objectives varying the number of items, *n* (size of search space is given by 2*n*) and the n feasibility ratio φ of the search space, which is a good indicator of the complexity of the landscapes in this kind of problems. This research work clearly shows that either convergence or diversity can be emphasized by contracting or expanding the dominance area. Also, the research work shows that the optimal value of $S*$ that controls the area of dominance depends strongly on all factors analysed here: number of objectives, size of the search space, and complexity of the problems.

Empirical Analysis

In this research work, we used the Deb-Thiele-Laumanns-Zitzler (DTLZ) Multi-objective problem to study and compare the effects on search performance of controlling dominance area of solutions. We also used the DZTL2 problem of the problem family with objectives {6, 8.}, n =100, 10000 items and user defined variable (*Si*) {0.45, 0.5, 0.65}

Experimental measures

In performing an analysis on the influence of control in the dominance area of the solutions, especially observing how the convergence and the diversity of the MODE algorithm are affected the following measures were used; Hyper volume, Generational Distance (GD), Inverse Generational Distance (IGD), and Spacing.

The hyper volume is used as a metric to evaluate sets of non-dominated solutions obtained by MOEAs. The hyper volume measures the m-dimensional volume of the region in objective space enclosed by the obtained non dominated solutions and a dominated reference point. The hyper volume is considered better when the set of non-dominated solution shows higher value. This happens from both convergence and diversity scenarios. In this, we used $(f1,f2,\ldots fm) = (0,0,0\ldots 0)$ as a reference point to calculate the hyper volume. The GD measures how far the generated approximated Pareto front PFapprox, i.e, the solutions generated by the CDASGDE algorithm, is from the true Pareto front of the problem. If GD is equal to 0 all points of PFapprox belong to the true Pareto front. With the GD we can observe if the algorithm converges for some region of the true Pareto Front. The IGD measures the minimum distance of each point of the true Pareto front to the points of the PFapprox. If IGD is equal to zero, the PFapprox contains every point of the true Pareto Front. With the IGD we can observe if the PFapprox converges to the true Pareto front and also if this set is well diversified. It is important to perform a joint analysis of these two indicators because with the analysis of GD we cannot identify if the solutions are distributed over the Pareto front, and only with IGD it is possible to define a sub-optimal solution as a good solution.

The other quality indicator used is the Spacing. This metric measures the range variance between neighbours' solution in the front. If the value of this metric is 0, all solutions are equivalently distributed in the objective space. For the comparison of the quality indicators, the differences among the results are defined by the Mann Whitney test. The Mann Whitney test is a non-parametric statistical test used to detect differences between algorithms. The test is applied to raw values of each metric.

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RESULTS / FINDINGS

This analysis was carried out using Generalized Differential Evolution on pareto dominance and using Control Dominance Area of Solutions on GDE **(CDASGDE)**. In these experiments, we show the average performance 10 runs, each of which spent on 10000 generations, and the population size is set to 100 with m{3,6,8}*on Si* (0.25, 0.45,0.5,0.65)

Table 1: 8 Objectives *Si* 0.65

HYPERVOLUME (CDASGDE AND GDE3)	GENERATIONAL DISTANCE (CDASGDE AND GDE3)	INVERTED GENERATIONAL DISTANCE (CDASGDE AND GDE3)	SPACING (CDASGDE AND GDE3)
Min: 0.0 Min: 0.0	Min:0.2800089005 Min: 0.138080992	Min:1.1508473171 Min: 1.0357960345	Min:0.974895622 Min: 0.679241167
Median: 0.0 Median: 0.0	Median:0.29707588 Median:0.14046691	Median:1.36183888 Median: 1.04970161	Median:1.00421087M edian: 0.76044690
Max:1.129925898 Max:0.006862412	Max:0.3024093378 Max: 0.1703482608	Max:1.6330432233 Max: 1.343819420	Max:1.04991339833 Max: 0.8037113369
Count: 5 Count: 5	Count: 5 Count: 5	Count: 5 Count: 5	Count: 5 Count: 5
Indifferent: [GDE3] Indifferent: [CDASGDE]	Indifferent: [] Indifferent:[]	Indifferent: [GDE3] Indifferent: [CDASGDE]	Indifferent: [] Indifferent:[]

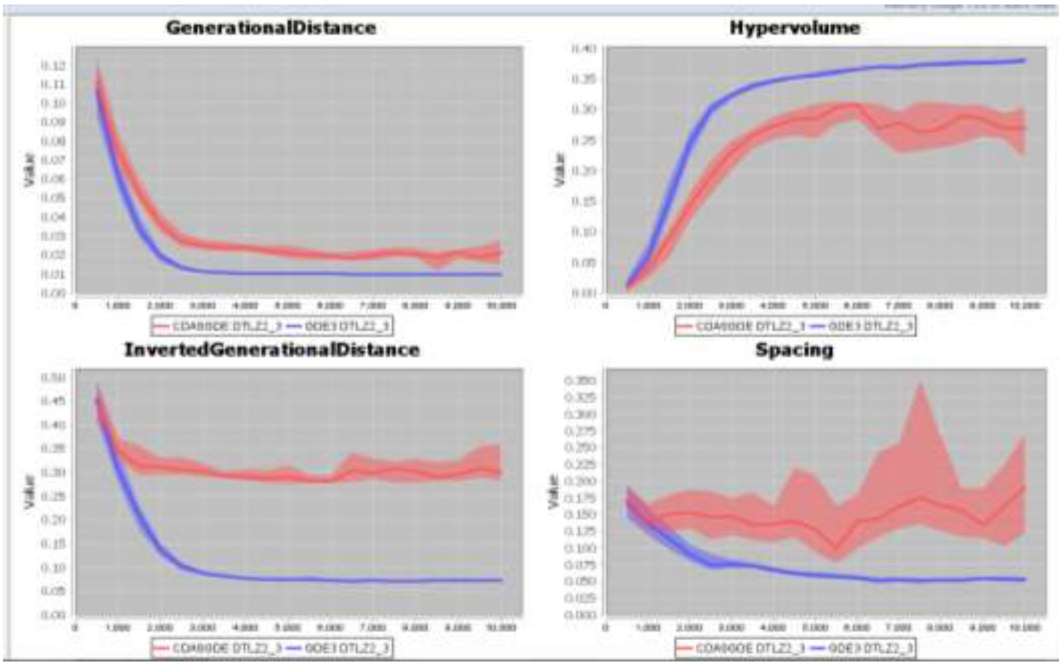

Figure 3: *Si*= 0.45 for 3 objectives

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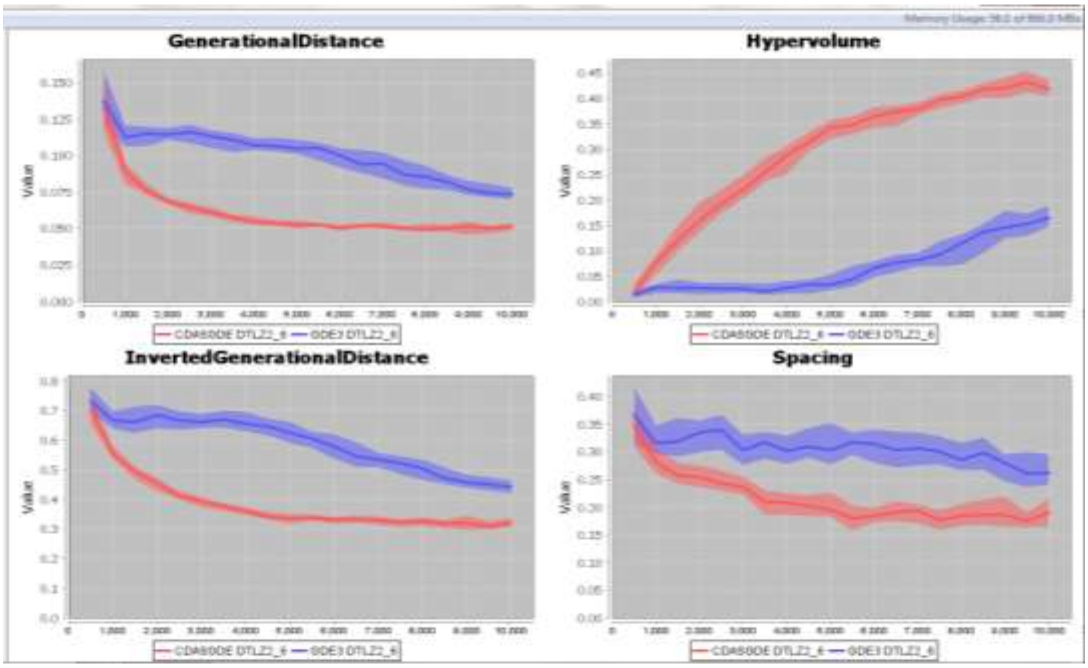

Figure 4: *Si*= 0.45 for 6 objectives

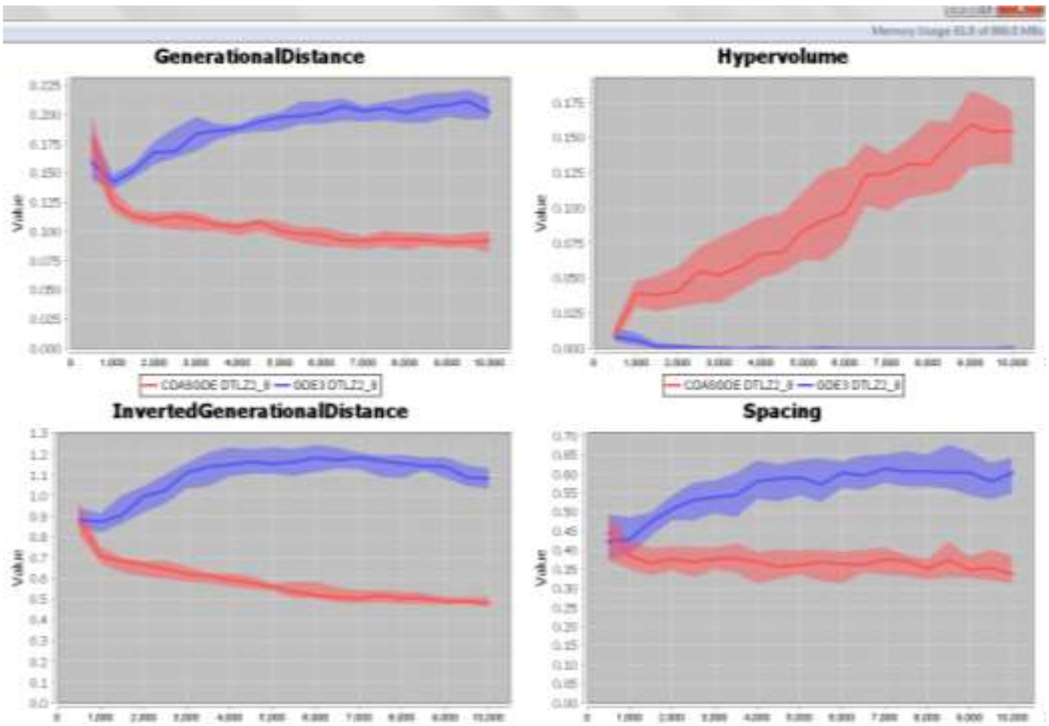

Figure 5: *Si*= 0.45 for 8 objectives

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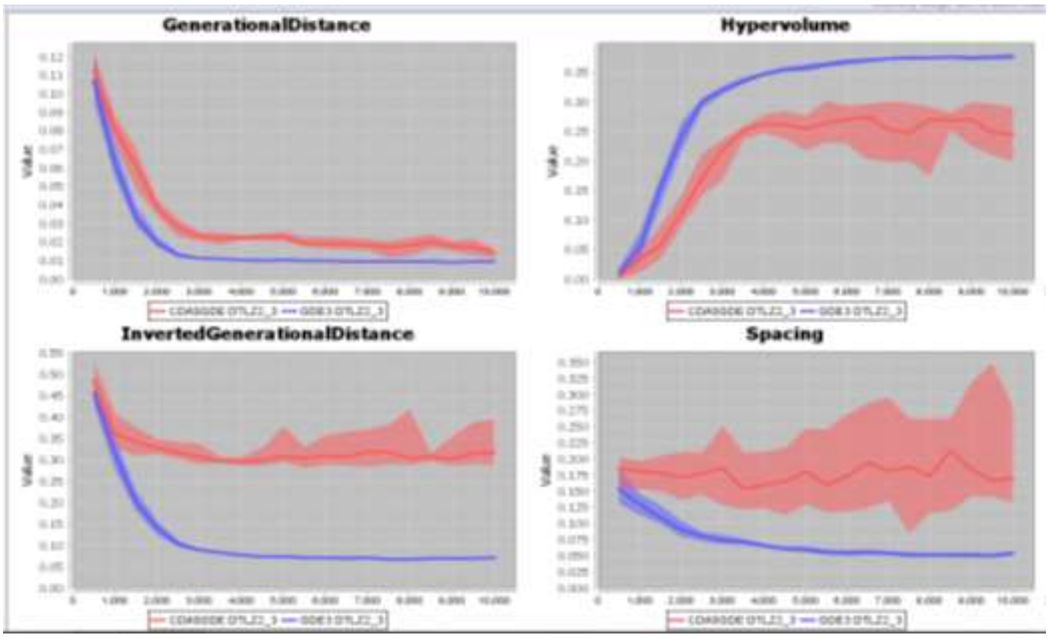

Figure 6: *Si*= 0.25 for 3 objectives

DISCUSSION

From the analysis shown in the Table 1, it can be seen that the CDASGDE algorithm performed better especially as the number of objectives increases. It does its evaluations using the four metrics. This was analyzed using the Mann Whitney nonparametric statistic test. With the values of the Mean, Median, Max, Count and indifferent. In situations where the indifferent had any of the algorithm enclosed in any of the metrics, it means that particular algorithm is not different in performance with the other for that particular metrics

The configurations with *Si* = 0.45, obtained the best results for almost all objectives, i.e. obtained a significant difference with respect to the other configurations, 0.25, 0.5, 0.65. This was followed by 0.25, 0.65 then 0.5. When the number of objectives grew, it is observed that the convergence of the original GDE deteriorates, yielding poor GD values. These results stress the hypothesis, that the use of CDAS with Si <0.5 in DE improves the convergence of the algorithm to the true Pareto front. For the IGD, according to the Mann Whitney test, the results of the configurations with Si = 0.25, and 0.45 had the best values. When the number of objectives is small, GDE3 with the original Pareto dominance relation still has competitive IGD values; however, when this number grows its performance deteriorates. The configuration with Si = 6.5 again did not obtain the best values, however, some configurations had good values. With the joint examination of these two indicators it can be concluded that the CDAS with Si <0.5 produced very good results for many objectives than when *Si* = 0.5 or 0.65. In this situation, the generated PFapprox converges to the true Pareto Front, furthermore the PFapprox is diversified and cover almost all the true Pareto Front. The configurations with Si <0.5 provides more convergence and diversity than the original Pareto dominance relation. For the spacing indicator, the best configuration defined by the Mann Whitney test is Si = 0.25. However, this occurs because for almost all the objectives this configuration generated only one solution in the PFapprox. Again, the configuration with the original Pareto dominance relation obtained the worst values. The configurations with Si = 0.45 obtained good spacing

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values due to the low number of solutions in the PFapprox. Here, the use of CDAS with Si < 0.5 in GDE diversify the search and help the algorithm to produce a well-distributed PFapprox.

CONCLUSION

In this research work, we presented a study of the influence of Controlling the Dominance Area of Solutions in a Multi-Objective Differential Evolution Algorithm. The concept of CDAS was observed and used; an empirical analysis was carried out to measure how the CDAS affects the convergence and diversity of a MODE algorithm in different multi-objective scenarios. The chosen MODE algorithm was the GDE3. It was investigated that the convergence and diversity aspects of the CDAS-GDE in two different situations, using Si <0.5 and Si >0.5, i.e. perform a relaxation of the Pareto dominance relation. The experiments were conducted with a multi-objective problem, DTLZ2, and the objectives were varied in three different values: 3, 6, and 8. The analysis of convergence and diversity of the algorithm was based on four quality indicators; hyper volume, generational distance, inverse generational distance and spacing. The Mann Whitney statistical test was used to detect difference between the configurations. Using the CDAS with Si <0.5 produced a PFapprox that converged to true Pareto Front and covered almost its entire area. Using the CDAS with Si >0:5 produced a PFapprox that did not converge to the true Pareto Front. For Si = 0.5 produced a diversified PFapprox for high degrees. However this diversified PFapprox do not converged to the true Pareto Front. The results produced by the original Pareto dominance relation deteriorate when the number of objectives grows and the algorithm generated sub-optimal solutions.

Future Research

With the experiment carried out and the results obtained; we have been able to contribute to knowledge by introducing CDAS into Differential Evolution algorithm as a way to bring about enhancement in its selection operator; induce appropriate ranking amongst solutions in situations where there are many multi-objectives which has really improved the performance of the algorithm. Future work should consider different selection operators / process and the results should be evaluated with that of this work for better efficiency and enhancement.

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