Isere, *Cogent Mathematics* (2016), 3: 1246074
http://dx.doi.org/10.1080/23311835.2016.1246074

cogent mathematics

Received: 27 July 2016
Accepted: 05 October 2016
First Published: 11 October 2016

*Corresponding author: A.O. Isere, Department of Mathematics, Ambrose Alli University, Ekpoma 310001, Nigeria
E-maills: abednis@yahoo.co.uk, isereao@aauekpoma.edu.ng

Reviewing editor:
Lishan Liu, Qufu Normal University, China

Additional information is available at the end of the article

PURE MATHEMATICS | RESEARCH ARTICLE

Natural rhotrix

A.O. Isere[1*]

Abstract: This paper takes a look at a set of rhotrix whose entries are ordered natural numbers. This rhotrix is called the natural rhotrix. Properties of this set are examined and the results are presented. Since the natural rhotrix R is not invertible, a maiden investigation is made into the concepts of minor rhotrices of R, determinant functions ($h(R)$), codeterminant function ($\text{codet}(R)$), and index (ρ) of a natural rhotrix. It was found that $\text{codet}(R) = \rho h(R)$, and their methods of computations are outlined for mathematical enrichment.

Subjects: Science; Mathematics & Statistics; Advanced Mathematics; Algebra

Keywords: rhotrix; determinant function; codeterminant function and index

2010 Mathematics subject classifications: primary 15B99; secondary 08-02

1. Introduction

A rhotrix A of dimension three is a rhomboidal array defined as:

$$A = \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle$$

where $a, b, c, d, e \in \Re$. The entry c in A is called the heart of A, denoted as $h(A)$. The concept of rhotrix was introduced by Ajibade (2003) as an extension of matrix-tertions and matrix-noitrets by Atanassov and Shannon (1998). The concept of rhotrices is still at its elementary stage of development. It is barely over a decade ago. Since its birth in 2003, many researchers have shown interest

ABOUT THE AUTHORS

A.O. Isere obtained first and second degrees in Mathematics from the University of Benin, Benin City, Nigeria, in 1999 and 2004, respectively. He was employed by Ambrose Alli University, Ekpoma, Nigeria in 2006, as a lecturer and a researcher in Mathematics till date. He got his PhD in Algebra from Federal University of Agriculture, Abeokuta, Nigeria, in 2014. He specializes in Loop Theory, precisely in Osborn loops. Between 2012 and 2015, he has published various articles on finite Osborn loops, Holomorphy of Osborn loops and Osborn loops of order $4n$.

In 2015, he picked interest in Rhotrix algebra and is currently working on the classical and non-classical rhotrices, and Rhotrix loops.

But in this article generalizes the examples of natural rhotrix, introduces the concept of determinant function, codeterminant function and index of natural rhotrix.

PUBLIC INTEREST STATEMENT

A natural rhotrix is a rhotrix whose elements are all countable numbers. A rhotrix is a system of numbers in a rhomboid shape. They are similar to matrices. These systems of numbers form sets of rhotrices depending on the amount of numbers in the systems. Each set obeys some mathematical rules depending on the type of numbers you have in the set and behave differently.

This work examines the behavior of a rhotrix set whose elements are countable numbers. In real life, children from different families behave differently. Therefore, a scientific study of each family will improve inter human relationship and help to harness the talents inherent in each of the family. This is to illustrate the essence of the scientific study of families or sets of physical properties around us like this work.

Detail introduction, transformation into matrices and applications abound in literature.

in developing and expanding this concept, most often, in analogy with the concepts of matrices usually through a transformation that converts a matrix into a rhotrix and vice versa (Ajibade, 2003; Sani, 2008). One of such works was the classification of rhotrices into sets and algebraic spaces by Mohammed and Tella (2012). The paper classifies rhotrices into natural rhotrix set, real rhotrix set, complex rhotrix set, rational and irrational rhotrix sets. Thus, their work has actually opened up different branches of studying rhotrices. Therefore, this article is picking on the first branch, the natural rhotrix set. Furthermore, in this paper, most of these properties of natural rhotrices will be examined without necessarily having to go through a transformation. Though, if need be, these properties would always be in conformity to one transformation or the other. This can be examined.

Definition 1.1 Mohammed and Tella (2012) A rhotrix set is called a natural rhotrix set if its rhotrice entries belong to the set of natural numbers. For example,

$$\hat{R_3}(\mathbb{N}) = \left\{ \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle : a, b, c, d, e \in \mathbb{N} \right\}$$

is the set of all three-dimensional natural rhotrices.

This set of natural rhotrix is a beautiful rhotrix with unique characteristics many of which are yet to be discovered.

In this work, we will be looking at some fundamental properties of this algebraic set.

Definition 1.2 Ajibade (2003), Mohammed and Tella (2012) A real rhotrix set of dimension three, denoted as $\hat{R_3}(\mathfrak{R})$ was defined by Ajibade as

$$\hat{R_3}(\mathfrak{R}) = \left\{ \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle : a, b, c, d, e \in \mathfrak{R} \right\}$$

where $h(R) = c$ is called the heart of any rhotrix R belonging to $\hat{R_3}(\mathfrak{R})$ and $\mathfrak{R}$ is the set of real numbers. Examples showing extension of this set and analysis are copious in literature (Aminu & Michael, 2015; Baumslag & Chandler, 1968; Ezugwu, Ajibade, & Mohammed, 2011; Mohammed, 2009, 2014; Mohammed, Balarabe, & Imam, 2012; Tudunkaya & Manjuola, 2010; Usaini & Mohmmed, 2012). It is worthy to note that an n-dimensional real heart-based rhotrix denoted by $\hat{R_n}(\mathfrak{R})$, will have it cardinality as $|\hat{R_n}(\mathfrak{R})| = \frac{1}{2}(n^2 + 1)$, where $n \in 2\mathbb{Z}^+ + 1$. This implies that all heart-based or heart-oriented rhotrices are of odd dimension (≥3). All operations (addition and multiplication) in this work, will be as defined by Ajibade in (2003). Thus, addition and multiplication of two heart-based rhotrices are defined as:

$$R + Q = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle + \left\langle \begin{array}{ccc} & f & \\ g & h(Q) & j \\ & k & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & a+f & \\ b+g & h(R)+h(Q) & d+j \\ & e+k & \end{array} \right\rangle$$

and

$$R \circ Q = \left\langle \begin{array}{ccc} & ah(Q)+fh(R) & \\ bh(Q)+gh(R) & h(R)h(Q) & dh(Q)+jh(R) \\ & eh(Q)+kh(R) & \end{array} \right\rangle$$

respectively. A generalization of this hearty multiplication is given in Mohammed (2014) and in Ezegwu et al. (2011), A row-column multiplication of heart-based rhotrices was proposed by Sani (2004) as:

$$R \circ Q = \left\langle \begin{array}{ccc} & af+dg & \\ bf+eg & h(R)h(Q) & aj+dk \\ & bj+ek & \end{array} \right\rangle$$

A generalization of this row-column multiplication was also later given by Sani (2007) as:

$$R \circ Q = \left\langle a_{i_1 j_1}, c_{i_2 j_2} \right\rangle \circ \left\langle b_{i_2 j_2}, d_{l_2 k_2} \right\rangle = \left\langle \sum_{i_2 j_1 = 1}^{t} (a_{i_1 j_1} b_{i_2 j_2}), \sum_{l_2 k_1 = 1}^{t-1} (c_{l_1 k_1} d_{l_2 k_2}) \right\rangle, t = (n+1)/2.$$

where R_n and Q_n are n-dimensional rhotrices (with n rows and n columns).

Definition 1.3 A rhotrix is said to be invertible if it has an inverse

For example let

$$A = \left\langle \begin{array}{ccc} & a & \\ b & h(A) & d \\ & e & \end{array} \right\rangle$$

Then, the inverse of A is given as

$$A^{-1} = -\frac{1}{(h(A))^2} \left\langle \begin{array}{ccc} & a & \\ b & -h(A) & d \\ & e & \end{array} \right\rangle$$

It is observed that any rhotrix $h(R) \neq 0$ is an invertible or non-singular rhotrix. However, a natural rhotrix is a singular rhotrix. That is, we cannot find A^{-1} for which A is a natural rhotrix.

2. Preliminaries

In this section, we shall strictly concern ourselves with the natural rhotrix—a rhotrix whose parent set is the set of well-ordered natural numbers. A natural rhotrix starts with a dimension one (i.e. R_1). Therefore, the cardinality of n-dimensional natural rhotrix is given by $|\hat{R}_n(\mathbb{N})| = \frac{1}{2}(n^2+1)$, where $n \in 2\mathbb{N}+1$.

Remark 2.1 Recall that the set $\mathbb{N} = \{0, 1, 2, 3, \dots\}$. see Aashikpelokhai, Agbeboh, Uzor, Elakhe, and Isere (2010), Baumslag and Chandler (1968)-The set of natural number or better still the set of non-negative integers. This set $2\mathbb{N}+1$ is larger than $2\mathbb{Z}^+ + 1$. Therefore, this paper is expanding the scope of set of rhotrice dimensions as hitherto presented in literature.

Definition 2.1 (Major row and major column) The major row and the major column are usually the only full row and full column in a rhotrix. They are usually at the middle of the rows and columns of any dimensional rhotrix.

A natural rhotrix, as other rhotrices, has one major row and one major column.

3. Examples of natural rhotrices

This section gives different representations of natural rhotrices as examples according to their dimensions.

(a) A natural rhotrix of dimension one (R_1) is given by:

$$R_1 = \langle a \rangle$$

where $a \in \mathbb{N}$. It only has a heart with no other entries.

(b) A natural rhotrix of dimension three (R_3) is given by:

$$R_3 = \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle$$

where $a, b, c, d, e \in \mathbb{N}$

Remark 3.1 All the entries are non-zero elements of $\mathbb{N}$. This remark holds for the other examples of natural rhotrices.

(c) A natural rhotrix of dimension five (R_5) is given by:

$$R_5 = \left\langle \begin{array}{ccccc} & & a & & \\ & b & c & d & \\ e & f & g & h & j \\ & k & l & m & \\ & & n & & \end{array} \right\rangle$$

where $a, b, c, d, e, f, g, h, j, k, l, m, n \in \mathbb{N}$

(d) A natural rhotrix of dimension seven (R_7) is given by:

$$R_7 = \left\langle \begin{array}{ccccccc} & & & a & & & \\ & & b & c & d & & \\ & e & f & g & h & i & \\ j & k & l & m & n & o & p \\ & q & r & s & t & u & \\ & & v & w & x & & \\ & & & y & & & \end{array} \right\rangle$$

where $a, b, c, \ldots, y \in \mathbb{N}$.

Remark 3.2 You can go on and on. For example, R_9 and R_{11} will have their last entries as 41 and 61, respectively.

(e) Generally, a natural rhotrix of dimension $n'(R_{n'})$ is given by:

$$R_{n'} = \left\langle \begin{array}{ccccccc} & & & a & & & \\ & & b & c & d & & \\ & - & - & - & - & - & \\ - & - & - & n^2+n+1 & - & - & - \\ & - & - & - & - & - & \\ & & 2n^2+2n-2 & 2n^2+2n-1 & 2n^2+2n & & \\ & & & 2n^2+2n+1 & & & \end{array} \right\rangle$$

where $a, b, c, \ldots, 2n^2+2n+1 \in \mathbb{N} \, \forall \, n' \in 2\mathbb{N}+1$ and $n \in \mathbb{N}$ (i.e. $n' = 2n+1$)

Remark 3.3 The above is the generalization of any natural rhotrix.

4. Properties of natural rhotrix

LEMMA 4.1 *Let R_i be any i dimensional natural rhotrix. Then, the heart $(h(R_i))$ is the middle value of a set of n numbers that make up the rhotrix if and only if $n = |R_i|$ $i = 1, 3, 5, \ldots$*

Proof First part

Since $n \in 2\mathbb{N} + 1$, then there exist meddle value (median).

So, if $n = |R_i|, 1, 3, 5, \ldots$ then for $i = 1$ is trivial.

So, $i = 3 \implies n = 5$ entries which are ordered natural numbers. Thus, the median is $3 = \frac{1}{2}(|R_3| + 1)$.

So, $i = 5 \implies n = 13$ entries which are ordered natural numbers. Thus, the median is $7 = \frac{1}{2}(|R_5| + 1)$.

So, $i = 2k + 1 \implies n = 2k^2 + 2k + 1$ entries which are ordered natural numbers. Thus, the median is $n^2 + n + 1 = \frac{1}{2}(|R_{2k+1}| + 1)$.

The converse follows from the Cardinality of R_n where $n \in 2\mathbb{N} + 1$.

THEOREM 4.1 *Let R_n be any n dimensional natural rhotrix. Then, the following are equivalent:*

(a) The cardinality $|R_n| = \frac{1}{2}(n^2 + 1)$ where $n \in 2\mathbb{N} + 1$
(b) The last entry will be the value $2n'^2 + 2n' + 1 \forall n' \in \mathbb{N}$
(c) The heart of $R_n[h(R_n)]$ is represented by $h = \frac{1}{2}(|R_n| + 1), n \in 2\mathbb{N} + 1$.
(d) The $h(R_n)$ will be the value $n'^2 + n' + 1$

Proof

(a) $\Rightarrow$ (b) Since $|R_n| = \frac{1}{2}(n^2 + 1)$ where $n \in 2\mathbb{N} + 1$, then for all $n' \in \mathbb{N}$, $n = 2n' + 1$. Then $|R(n' + 1)| = 2n'^2 + 2n' + 1$.

(b) $\Rightarrow$ (c) Since the last entry is $2n'^2 + 2n' + 1$ and is old, then by Lemma 4.1, the middle value is

$$\frac{2n'^2 + 2n' + 1}{2} + \frac{1}{2} = \frac{1}{2}(|R_n| + 1) = h(R_n) \, \forall \, n \in 2\mathbb{N} + 1$$

(c) $\Rightarrow$ (d) Given that

$$h(R_n) = \frac{1}{2}(|R_n| + 1) \, \forall \, n \in 2\mathbb{N} + 1$$

and letting $n = 2n' + 1$ gives

$$h(R_n) = n'^2 + n' + 1$$

(d) $\Rightarrow$ (a) Since $h(R_n) = n'^2 + n' + 1$ and by Lemma 4.1, $2h(R_n) = |R_n| + 1$, then

$$|R_n| = \frac{1}{2}(n^2 + 1)$$

Remark 4.1 Theorem 4.1 is simply a characterization of the natural rhotrix

4.1. Determinant function

Though, the natural rhotrices are not invertible rhotrices. However, the heart of a natural rhotrix plays the role of a determinant function.

LEMMA 4.2 *Let A and B be any natural rhotrices of dimension n and |A| a determinant function of A , then*

$|AB| = |A||B|$

Proof Let $|A| = h(A)$ and $|B| = h(B)$ then

$|AB| = |h(A)h(B)| = |A||B|$

Remark 4.2

(i) Again, the justaposition *AB* represents $A \circ B$ as defined in Ajibade (2003)
(ii) We will adopt $h(A)$ to mean the determinant function of *A*.

For example, consider a three-dimensional rhotrix (R_3):

$$A = \left\langle \begin{array}{ccc} & a & \\ b & h(A) & d \\ & e & \end{array} \right\rangle$$

The determinant function of the above rhotrix *A* is designated by h(A)

Computing the value of the determinant function of any natural rhotrix is simply the value of its heart. However, for higher natural rhotrices, we introduce the concept of minor rhotrices and codeterminant functions. That brings us to the next subsection.

4.2. Codeterminant function

The concept of codeterminant functions is similar to that of minor matrices. In natural rhotrices of dimension three (R_3), the codeterminant function is the same as the determinant function. But for higher natural rhotrices the codeterminant functions are not necessarily the same. To evaluate the codeterminant function of a higher natural rhotrix, we need to first split or reduce the higher natural rhotrix into chain(s) of R_3 called the minor rhotrices and the determinant function of each of them is evaluated as above. These minor rhotrices are split or reduced either along the major column or along the major row and their determinant functions are summed up accordingly. For a natural rhotrix with well-ordered entries, the result is the same regardless of whether it is summed up along the major column or along the major row.

Consider the natural rhotrix of dimension five below:

$$A = \left\langle \begin{array}{ccccc} & & a & & \\ & b & c & d & \\ e & f & g & h & j \\ & k & l & m & \\ & & n & & \end{array} \right\rangle$$

Behold, $h(A) = g$.

To evaluate $\text{codet}(A)$, the higher rhotrix needs to be split into minor rhotrices of dimension three.

$$\text{codet}(A) = \left\langle \begin{array}{ccccc} & & a & & \\ & b & c & d & \\ e & f & g & h & j \\ & k & l & m & \\ & & n & & \end{array} \right\rangle = c + l$$

That is splitting the rhotrix *A* along the major column gives:

$$\operatorname{codet}(A) = \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & g & \end{array} \right\rangle + \left\langle \begin{array}{ccc} & g & \\ k & l & m \\ & n & \end{array} \right\rangle = c + l$$

While splitting along the major row gives:

$$\operatorname{codet}(A) = \left\langle \begin{array}{ccc} & b & \\ e & f & g \\ & k & \end{array} \right\rangle + \left\langle \begin{array}{ccc} & d & \\ g & h & j \\ & m & \end{array} \right\rangle = f + h$$

Remark 4.3 For a natural rhotrix $c + l = f + h$.

Example 4.1 Find the determinant and the codeterminant functions of the natural rhotrix below:

$$A = \left\langle \begin{array}{ccccc} & & 1 & & \\ & 2 & 3 & 4 & \\ 5 & 6 & 7 & 8 & 9 \\ & 10 & 11 & 12 & \\ & & 13 & & \end{array} \right\rangle$$

Solution

$$h(A) = 7$$

Next, we find codeterminant function, first along the major column gives:

$$\operatorname{codet}(A) = \left\langle \begin{array}{ccc} & 1 & \\ 2 & 3 & 4 \\ & 7 & \end{array} \right\rangle + \left\langle \begin{array}{ccc} & 7 & \\ 10 & 11 & 12 \\ & 13 & \end{array} \right\rangle = 3 + 11 = 14$$

Now along the major row gives:

$$\operatorname{codet}(A) = \left\langle \begin{array}{ccc} & 2 & \\ 5 & 6 & 7 \\ & 10 & \end{array} \right\rangle + \left\langle \begin{array}{ccc} & 4 & \\ 7 & 8 & 9 \\ & 12 & \end{array} \right\rangle = 6 + 8 = 14$$

Extension to higher dimension can be made in a similar manner used in reducing R_5 to minor rhotrices of R_3. Let us consider the next example.

Example 4.2 Find the determinant and the codeterminant functions of the following: (i) along major column (ii) along the major row of the rhotrix below:

$$Q = \left\langle \begin{array}{ccccccc} & & 1 & & & & \\ & & 2 & 3 & 4 & & \\ & 5 & 6 & 7 & 8 & 9 & \\ 10 & 11 & 12 & 13 & 14 & 15 & 16 \\ & 17 & 18 & 19 & 20 & 21 & \\ & & 22 & 23 & 24 & & \\ & & 25 & & & & \end{array} \right\rangle$$

Solution

$$h(Q) = 13$$

(i)

$$\operatorname{codet}(Q)=\left\langle \begin{array}{ccc} & 1 & \\ 2 & 3 & 4 \\ & 7 & \end{array} \right\rangle+\left\langle \begin{array}{ccc} & 7 & \\ 12 & 13 & 14 \\ & 19 & \end{array} \right\rangle+\left\langle \begin{array}{ccc} & 19 & \\ 22 & 23 & 24 \\ & 25 & \end{array} \right\rangle=3+13+23=39$$

(ii)

$$\operatorname{codet}(Q)=\left\langle \begin{array}{ccc} & 5 & \\ 10 & 11 & 12 \\ & 17 & \end{array} \right\rangle+\left\langle \begin{array}{ccc} & 7 & \\ 12 & 13 & 14 \\ & 19 & \end{array} \right\rangle+\left\langle \begin{array}{ccc} & 9 & \\ 4 & 15 & 16 \\ & 21 & \end{array} \right\rangle=11+13+15=39$$

It is of utmost importance that the reader practices evaluating the codeterminant functions of higher rhotrices along major row or major column. This will give the reader the prerequisite confidence in obtaining the values of codeterminant functions in certain cases where a particular major row or column tends to accelerate the release of results which consequently leads to reduced hardship and computation time expended. Considering the Example 4.2, summing along the column seems to make life easier for the reader. However, the beauty of this work lies in its simplicity.

4.3. Index of rhotrices

The index of a natural rhotrix A is the number of minor rhotrices of dimension three that can be derived, either along the major column or along the major row, from A. This index is a whole number or better still a natural number. For example the index of R_3 is 1 and of R_5, R_7, and R_9 are 2, 3, and 4, respectively. Appropriately, the index of R_1 is zero.

THEOREM 4.2 *Given any rhotrix R, the* $\operatorname{codet}(R)=\rho h(R)$ *where ρ is a natural number called the index of R.*

Proof We prove using mathematical induction. Since an index of a natural rhotrix is a natural number corresponding to the number of R_3 that can be derived from R_n, $n \geq 3$, and $n \in 2\mathbb{N}+1$.

Now, when $n=3$, the

$$\operatorname{codet}(R_3)=h(R_3)$$

since the $\operatorname{codet}(R_3)$ is necessarily the $\det(R_3)=h(R_3)$. By Lemma 4.1. Implies that $\rho=1$, So, the equation is true for $n=3$.

For $n=5$, then we have two minors of R_3. That is,

$$\operatorname{codet}(R_5)=2h(R_5)$$

implies that $\rho=2$. So, the equation is true for $n=5$.

For $n=7$, then, we have three minors of R_3, then

$$\operatorname{codet}(R_7)=3h(R_7)$$

So, the equation is true for $n=7$ and $\rho=3$.

Then, for $n=2k+1$,

$$\operatorname{codet}(R_{2k+1})=kh(R_{2k+1})$$

Then, it is true for $n=2k+1$ and $\rho=k$.

For $n = 2k + 3$,

$$\operatorname{codet}(R_{2k+3}) = \operatorname{codet}(R_{2(k+1)+1}) = k + 1h(R_{2(k+1)+1})$$

Then, it is true for $n = 2k + 3$ and $\rho = k + 1$. Hence, the equation is true for all value of $n \geq 3$ and ρ a natural number.

THEOREM 4.3 *Giving any natural rhotrix R*

$$\operatorname{codet}(R) = \frac{\rho}{2}(|R| + 1)$$

where ρ is the index and $|R|$ is the cardinality of R, and $|R_n| = \frac{1}{2}(n^2 + 1)$

Proof Since $\operatorname{codet}(R) = \rho h(R)$ and by Lemma 4.1, determinant function is $h(R)$. Then, the result follows from the Theorem 4.1.

5. Conclusion

This article examined the properties of the natural rhotrix set, introduced the concepts of minor rhotrices, determinant functions, codeterminant functions and index of natural rhotrices. These concepts are a novelty to rhotrix algebra, and their methods of computations are presented for mathematical enrichment. Using the results in this paper, one can evaluate the determinant function of any n-dimensional rhotrix at a glance. With the links between the determinant functions, codeterminant functions and the index one will be able do a sketch of any n-dimensional natural rhotrix no matter how large is the value of n.

Acknowledgements
The author wishes to express his profound gratitude and appreciation to Dr Abdul Mohammed of Ahmadu Bello University, Zaria for his encouragement and valuable discussion, and to the various anonymous reviewers whose contributions have helped to improve this work.

Funding
The author received no direct funding for this research.

Author details
A.O. Isere[1]
E-mails: abednis@yahoo.co.uk, isereao@aauekpoma.edu.ng
[1] Department of Mathematics, Ambrose Alli University, Ekpoma 310001, Nigeria.

Citation information
Cite this article as: Natural rhotrix, A.O. Isere, *Cogent Mathematics* (2016), 3: 1246074.

References

Aashikpelokhai, U. S. U., Agbeboh, G. U., Uzor, J. E., Elakhe, A. O., & Isere, A. O. (2010). *A first course in linear algebra and algebraic strutures*. PON Publishers.

Ajibade, A. O. (2003). The concept of rhotrix in mathematical enrichment. *International Education in Science and Technology, 34*, 175–177.

Aminu, A., & Michael, O. (2015). An introduction to the concept of paraletrix, a generalization of rhotrix. *Journal of the African Mathematical Union and Springer-Verlag, 26*, 871–885.

Atanassov, K. T., & Shannon, A. G. (1998). Matrix-tertions and matrix-noitrets: Exercise for mathematical enrichment. *International Education in Science and Technology, 29*, 898–903.

Baumslag, B., & Chandler, B. (1968). *Theory and problems of group theory*. Schaum's outline series.

Ezugwu, E. A., Ajibade, A. O., & Mohammed, A. (2011). Generalization of heart-oriented rhotrix multiplication and its algorithm implementation. *International Journal of Computer Applications, 13*, 5–11.

Mohammed, A. (2009). A remark on the classifications of rhotrices as abstract strutures. *International Journal of Physical Sciences, 4*, 496–499.

Mohammed, A. (2014). *A new expression for rhotrix advances in linear algebra & matrix theory, 4*, 128–133.

Mohammed, A., Balarabe, M., & Imam, A. T. (2012). Rhotrix linear transformation. *Advances in Linear Algebra & Matrix Theory, 2*, 43–47.

Mohammed, A., & Tella, Y. (2012). Rhotrix sets and rhotrix spaces category. *International Journal of Mathematics and Computational Methods in Science and Technology, 2*, 2012.

Sani, B. (2004). An alternative method for multiplication of rhotrices. *International Journal of Mathematical Education in Science and Technology, 35*, 777–781.

Sani, B. (2007). The row-column multiplication for higher dimensional rhotrices. *International Journal of Mathematical Education in Science and Technology, 38*, 657–662.

Sani, B. (2008). Conversion of a rhotrix to a coupled matrix. *International Journal of Mathematical Education in Science and Technology, 39*, 244–249.

Tudunkaya, S. M., & Manjuola, S. O. (2010). Rhotrices and the construction of finite fields. *Bulletin of Pure and Applied Sciences, 29*, 225–229.

Usaini, S., & Mohmmed, L. (2012). On the rhotrix eigenvalues and eigenvectors. *Journal of the African Mathematical Union and Springer-Verlag, 25*, 223–235.