Journal of Advances in Science and Engineering, ISSN: 2636-607X
Volume 1 Number 2 (2018): 37-46

Journal of Advances in Science and Engineering (JASE)

Robust Stabilisation of Reformer Coupled Tanks

[1*]Odiamenhi, A. Martins, [2]Aigboje, O. Eddy, & [3]Ugboya, A. Paul

[1,2,3] *Department of Industrial and Production Engineering, Ambrose Alli University, Ekpoma, Nigeria,*

*Corresponding author's e-mail: odiamenhimartins@yahoo.com

Abstract

Diesel plays a significant role in the energy consumption in most developing countries. It serves as a source of power for the transportation, agricultural and industrial sectors. Diesel is one of the product of the petroleum, being utilised in all types of compression ignition engines as fuel. Hence, the need for reformer tank to manage proper waste is needed to avoid economical loss and environmental damage. The utilisation of reformer tanks in the management of waste energy is of economic and environmental advantage to the nation. This paper presents the use of feedback linearization and back-stepping methods to control the nonlinear reformers tanks in order to achieve robust control and stabilisation of the diesel. The model was transformed to a motion control model and the efficiency of the suggested algorithm was tested. The result showed that the back-stepping controller design is satisfactory not only for the tracking performance but also for the determination of the stability region.

Keywords

Back-stepping algorithm;
Diesel;
Feedback linearization;
Lyapunov function;
Reformers tanks;
Waste energy.

Received 10 August 2018; Revised 8 Sept. 2018; Accepted 9 Sept. 2018; Available online 15 Sept. 2018.

1. Introduction

Reformer is a machine that converts waste motor oil into diesel fuel that can run any diesel engine. A functioning engine needs lubrications or lubricating (engine) oil are used for lubrication. After a given duration, these used engine oils are taken out. During lubrication process, only about 20% of the oil are consumed while the remaining 80% act as impurities, thus a large quantity of the engine oil is left as wastage from different transport sectors every day (Beg *et al.*, 2010; Naima & Liazid, 2013).

Diesel is one of the product of the petroleum utilised in all kinds of compression ignition engines as fuel (Beg *et al.*, 2010). In other words, it plays a major role in the energy consumption of most developing countries, serving as a source of fuel for agricultural, transportation and industrial sectors (Okundamiya *et al.,* 2014). Considering waste lubricant as an alternative has multiple benefits like utilisation of waste energy source, thereby protecting the environment from toxic and hazardous chemical, reducing dependence on fossil fuel and provides economic advantage to the nation by increasing their foreign exchange (Bacha *et al.*, 2007).

Reformers make use of coupled tanks in processing the waste oil to diesel and the need for the proper modelling and control of these tanks cannot be over emphasised in the petroleum industries. The dynamics of the reformer coupled tanks are non-linear mathematical differential equation, therefore the need to determine the stability and controllers to achieve the desired goal of converting waste oil to diesel fuel.

Various tools of nonlinear control design are being used for the design of the system and controllers such as integral control, linearization, gain scheduling, back-stepping, feedback linearization, passivity based control and high gain observers (Khalid, 2002; Uswarman *et al.*, 2013).

Nail *et al.* (2015) used sliding mode control and optimal algorithm to determine the performance and optimal point of the coupled tank. John *et al.* (2015) modelled and controlled the coupled tank liquid level system using back-stepping method. Bastida *et al.* (2013) developed a model for controlling coupled tanks by using Labview. Boubakir *et al.* (2009) developed a controller for tanks using Neuro-fuzzy-sliding mode method. This work focused on the application of artificial intelligent and sliding mode on tank in other to achieve the desired goal. There is a drawback in reformer tank control to overcome difficulties like nonlinearities, uncertainties and

Article citation: Odiamenhi, A. M., Aigboje, O. E., & Ugboya, A. P. (2018). Robust Stabilisation of Reformer Coupled Tanks. *Journal of Advances in Science and Engineering, 1*(2): 37-46. Retrieved from https://www.sciengtexopen.org/index.php/jase/article/view/23

coupling from different aspects (Soltanpour & Fateh, 2009). In this paper, a robust stabilisation under uncertainties with a compromise between bounds of uncertainties and tracking performance is considered. This method presents a uniform bounded error convergence in the case of a broad range of ambiguities and the use of the Laypunov based theory guarantees stability of nonlinear system.

2. Methodology

This section describes the tank model and computational tool, which are feedback linearization and back-stepping method.

2.1. Tank Model

The schematic shown in Figure1 represents the model of a two degree of freedom state coupled tank system (Slotine & Li, 1991).

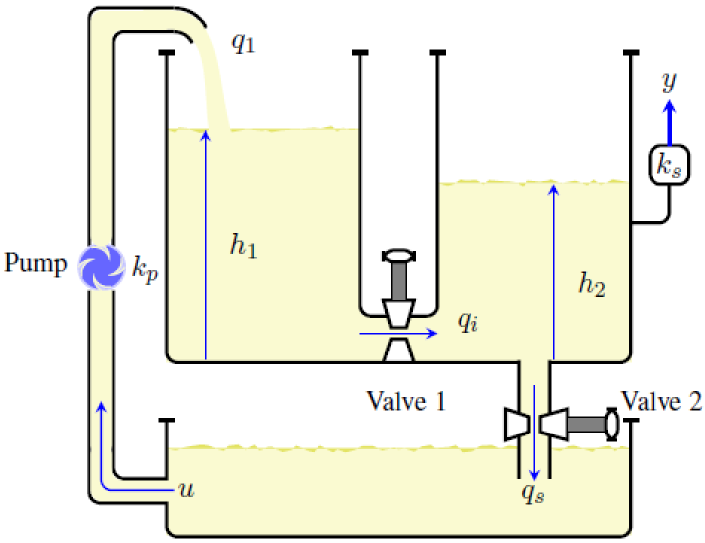

Figure 1. A coupled tank system

The coupled tank system is described by the second order non-linear differential equation

$$\left.\begin{aligned} \frac{dh_1}{dt} &= 1/C(-S_1 a_1\sqrt{2g(h_1-h_2)}) + k_p U \\ \frac{dh_2}{dt} &= 1/C(-S_1 a_1\sqrt{2g(h_1-h_2)} - S_2 a_s\sqrt{2g h_2}) \\ \mathrm{y} &= K_s h_2 \end{aligned}\right\} \tag{1}$$

With h_1 and h_2 as the liquid level in the tank respectively, g is the gravitational constant, S is the channel area, and k_p and k_s are pump and sensor gains.

2.2. Feedback Linearization

The tool of nonlinear system transforms (1) into an equivalent linear system, which could be either of the following (Kannan, 2012):

(a) Full state linearization- the state equation is completely linearized.

(b) Input-output linearization- where the input- output map is linearized, while the state equation can only be partially linearized

Full State Linearization: The full state equation is deduced as follows (Khalid, 2002):

$$\left.\begin{aligned} \dot{x}_n &= f_n(x_n) + \mathrm{G}(x_n)U \\ \mathrm{y} &= h_n(x_n), \\ U &= \alpha(x) + \beta(x)v \end{aligned}\right\} \tag{2}$$

where, v is the external reference input and a change of variable

$$Z = T(x) \tag{3}$$

ISSN: 2636-607X

where $f: D \rightarrow R^n$ *and* $G: D \rightarrow R$ are sufficiently smooth on a domain $D \subset R^n$ and is said to be feedback linearizable (input –state linearizable) if there exist a diffeomorphism $T: D \rightarrow R^n$ such that the $D_z = T(D)$ contain the origin and the change of variable.

$$\dot{x} = f(x) + G(x)U \tag{4}$$

$$y = h(x) \tag{5}$$

$$U = \alpha(x) + \beta(x)v \tag{6}$$

$$Z = T(x) \tag{7}$$

$$\dot{x} = f(x) + G(\dot{x})[\alpha(x) + \beta(x)v \tag{8}$$

$$x = [x_1, x_2]^T = [h_2, h_1]^T, \tag{9}$$

Such as:

$$\left.\begin{aligned} \dot{x_1} &= f_1(x) \\ \dot{x_2} &= f_2(x) + K_a U \\ y &= K_s x_1 \end{aligned}\right\} \tag{10a}$$

With,

$$\left.\begin{aligned} f_1(x) &= J_1\sqrt{(x_1 - x_2)} + J_2\sqrt{x_1} \\ f_2(x) &= -J_1\sqrt{(x_1 - x_2)} \end{aligned}\right\} \tag{10b}$$

$$J_1 = S_1 a_1 \sqrt{2g}/C, \quad J_2 = S_2 a_2 \sqrt{2g}/C, \; K_a = K_p/C$$

Where,

$$f(x) \triangleq \frac{f_1(x)}{f_2(x)},$$

$$g(x) \triangleq \begin{matrix} 0 \\ K_a \end{matrix}$$

$$y = K_s x_1 \tag{11}$$

$$Z = T(x) \tag{12}$$

Transforming the system (2) into the form (12):

$$\dot{Z} = AZ + B\gamma(x)[U - \alpha(x)]. \tag{13}$$

With (A,B) controllable and $\gamma(x)$ non singular for all $x \in D$, (A,B) is said simply to be completely state controllable if and only if there exist a control input $U(t)$ that will drive the initial state $x(t_0)$ at initial time, $t_0$to any desired final state over a finite time interval. There are two method used in determining whether a system is completely controllable or not, namely, *Karman's* method and *Gilbert's* method. In this paper, only the Karman method is explained because MATLAB implementation of controllability is based on this method (Ogata, 2002).

$$\emptyset_c = [B \,\vdots\, AB \,\vdots\, A^2B \,\vdots\, \cdots\cdots\cdots A^{n-1}B]. \tag{14}$$

Where *B* is the input matrix of the state space representation of the control system, *A* is the system matrix and n is the order of the controllability. The system is controllable if and only if the rank: $\emptyset_c = n$. This implies it is of full rank. Assume the system:

$$\left.\begin{aligned} \dot{x}_1 &= b sin x_2 \\ \dot{x}_2 &= -x^2{}_1 + U \\ y &= x_2 \end{aligned}\right\} \tag{15}$$

Transforming the *x* coordinate to *z* coordinate:

$$z_1 = x_2 \tag{16}$$

$$\dot{z}_1 = \dot{x}_1 = b sin x_2 = z_2 \tag{17}$$

$$\dot{z}_2 = b x_2 cos(\dot{x}_2) = b cos x_2(-x^2{}_1 + U) \tag{18}$$

and

$$U = -x^2{}_1 + (1/b cos x_2)v \tag{19}$$

$$\dot{z}_1 = z_2 \tag{20}$$

$$\dot{z}_2 = b cos x_2\left[-x^2{}_1 + x^2{}_1 + \left(\frac{1}{b cos x_2}\right)v\right) \tag{21}$$

$$\dot{z}_2 = v \tag{22}$$

$$y = x_2 = \sin^{-1}(z_2/b) \tag{23}$$

This method differs entirely from conventional (Jacobean) linearization because full state linearization is achieved by exact state transformation and feedback rather than by linear approximation of the dynamics.

Input- Output Linearization: Consider the system (2):

$\dot{x}_n = f_n(x_n) + G(x_n)U$

$y = h_n(x_n)$

Given a scalar function $h_n(x_n)$ and vector field $f_n(x_n)$; where *f, g,* and *h* are sufficiently smooth in a domain $D \in R^n$, the mapping $f: D \to R^n$ and $g: D \to R^n$ are called vector field on *D*. The derivative *y* is given by the equation:

$$\dot{y} = \frac{dh}{dx}[f(x) + g(x)U] \stackrel{\text{def}}{=} L_f h(x) + L_g h(x)U \tag{24}$$

$L_f h(x) = \frac{dh}{dx} f(x), \ \frac{dh}{dx} g(x) = L_g h(x)U$,

Let the second tank h_2 as the controlled output rather than the position control, $\dot{y} = L_f h(x) + L_g h(x)U$ is called the lie derivative of *h* with respect to *f* or along *f*.

$L_g L_f h(x) = \frac{dL_f}{dx} h(x)$, $L_f^2 h(x) = \frac{dL_f}{dx} f(x)$

If $L_g h(x) = 0$, then $\dot{y} = L_f h(x)$, which is independent of *U* if we continue to calculate the second derivative of *y,* denoted by y_2, we obtain:

$$\left.\begin{aligned} \ddot{y} &= \frac{d(L_f h)}{dx}[f(x) + g(x)U] \\ \ddot{y} &= L_f^2 h(x) + L_g L_f h(x)U \end{aligned}\right\} \tag{25}$$

Then *U* does not appear in the equation of $y, \dot{y}, and\ \dot{y}^{(2)}$ with a nonzero coefficient;

$y^{(2)} = L_f^2 h(x) + L_g L_f h(x)U$, with output $y = x_1$, calculate the derivative of the output of (10), we obtain:

$$\dot{y} = K_s \dot{x}_1 = K_s f_1(x) = \frac{dx_1}{dx}[\dot{x}_1] \tag{26}$$

$$= K_s[J_1\sqrt{(x_2} - x_1) - J_2(x_1)^{0.5}]$$

$$L_g L_f h(x)U = [-\frac{J_2(x_2 - x_1)^{-\frac{1}{2}}}{2}]K_s K_a U \tag{27}$$

$$L_f^2 h(x) = K_s[-\frac{J_1(x_2 - x_1)^{-\frac{1}{2}}}{2}] \tag{28}$$

$$\ddot{y} = K_s\left[-\frac{J_1(x_2 - x_1)^{-\frac{1}{2}}}{2}\right] + [\frac{J_2(x_2 - x_1)^{-\frac{1}{2}}}{2}]K_s K_a U \tag{29}$$

The forgoing equation shows that the system is input – output linearizable since the state feedback control

$$U = 1/L_g L_f^{\rho-1} h(x)[-L_f^{\rho} h(x) + v(t)]. \tag{30}$$

This reduces the input – output map to:

$$y^{(\rho)} = v(t) \tag{31}$$

The equation (31) has relative degree of freedom of two in R^2, since in this case (the chain of $\rho = 2$ integrator)

2.3. Stabilisation of the System Using Back-stepping Method

Back-stepping is a technique developed by Kototovic (1992) for designing stabilising controls for a special class of nonlinear dynamical systems. These systems are fabricated from subsystems that radiate from an irreducible subsystem, which can be stabilised using other techniques. Nevertheless, because of this recursive structure, designers typically begin the design process at the known-stable system and "back out" new controllers, which progressively stabilise each outer subsystem. The process terminates when the final external control is reached. Hence, this process is known as back-stepping (Slotine & Li, 1991).

This paper examined the case of system described by equation having an upper triangular structure as follows:

$$\left.\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2 \dots x_n, U) \\ \dot{x}_2 &= f_2(x_2, x_3 \dots x_n, U) \\ \dot{x}_{n-1} &= f_{n-1}(x_{n-1} \dots x_n, U) \\ \dot{x}_n &= f_n(x_n, U) \end{aligned}\right\} \tag{32}$$

The function $f_1(x_1, \dots, x_{i+1} \dots x_n, U)$ are supposed to satisfy appropriate hypothesis. These systems are often referred to as systems in feed forward form. In consideration of the fact that they correspond to a cascade interconnection of n subsystem, starting with the lower subsystem and ending with the upper subsystem in which the i^{th} subsystem is feed by the output $x_{n+1} \dots x_n$ of all previous subsystem in the cascade. Because of this triangular structure, the design of stabilizing feedback can be achieved in a recursive way, based on the following procedure.

Suppose that a feedback law:$U_n = a_n(x)$ is known which stabilises the lower subsystem of the chain; then:

$$\dot{x}_n = f_n(x_n, U) \tag{33}$$

And set

$$U = a_n(x_n) + U_n \tag{34}$$

This yield a system described by equation whose structure is the same as the structure of (1) with U replaced by U_n, but in addition the subsystem corresponding to the last equation of the chain is stable, by construction if $U_n = 0$. Consider now the subsystem formed by (33) and (34), which have the form:

$$\dot{x}_{n-1} = f_{n-1}(x_{n-1}, x_n, \alpha_n(x) + U_n = f_{n-1}(x_{n-1}, x_n, U_n) \tag{35}$$

$$\dot{x}_n = f_n(x_n, \alpha_n(x) + U_n) = f_n(x_n, U) \tag{36}$$

Suppose the system design obeys the feedback law: $U_n = a_{n-1}(x_n, x_{n-1})$, which render it stable, then:

$$U_n = X_{n-1}(x_n, x_{n-1}) + U_{n-1} \tag{37}$$

This actually is the same as setting

$$U = X_n(x_n) + X_{n-1}(x_n, x_{n-1}) + U_{n-1} \tag{38}$$

On the original system (1.1) by driving this, a system is obtained in which the upper n-2 equation were U replaced by U_{n-1} and in which the subsystem corresponding to the lower two equations of the chains is stable, by construction, if $U_{n-1} = 0$.At this point, the last two equation of the chain can be considered as a single subsystem, with state $U_{n-1} = (x_n, x_{n-1})$ and input: U_{n-1}, which is stable when $U_{n-1} = 0$.

Consider the system:

$$\left.\begin{aligned} \dot{x}_1 &= x_1 - x_2(x_2^2 + x_3^2) + x_4 + U \\ \dot{x}_2 &= -x_2 - x_3(x_2^2 + x_3) + x_4 + U \\ \dot{x}_3 &= -x_3 - x_4 + U \\ \dot{x}_4 &= -x_3 - x_4 + U \end{aligned}\right\} \tag{39}$$

Preliminary control,

$$U = -x_3 - x_4 + U_1$$

$$\dot{x}_4 = -2x_4 + U_1$$

The Laypunov function yielding locally exponential stability (LES) and globally exponential stability (GES) of x_4 subsystem can be taken as:

$$V(x_4) = \frac{x_4^2}{2},$$

$$input\ U_1 = 0, \dot{x}_4 = -2x_4^2$$

Start with the lower subsystem Σ_3 and select U such that we can achieve asymptotical stability using a laypunov function.

$$\dot{V}(x) < 0$$

$$V(x) = 0.5x_1^2 + 0.5x_2^2 + 0.5x_3^2 + 0.5x_4^2 \tag{40}$$

$$\frac{\partial V}{\partial x_1} = x_1, \frac{\partial V}{\partial x_2} = x_2, \frac{\partial V}{\partial x_3} = x_3, \frac{\partial V}{\partial x_4} = x_4,$$

Suppose the feedback law:

$$U = -x_3 - x_4 \tag{41}$$

$$\dot{x}_3 = -x_3 - x_4 + U = -x_3 - x_4 - x_3 + x_4 \tag{42}$$

$$\dot{x}_3 = -2x_3 \tag{43}$$

$$\dot{V}(x_3) = \frac{\partial V}{\partial x_3} f(\dot{x}_3) \tag{44}$$

$$= -2x_3(x_3) = -2x_3^2$$

Define the error variable in the control as U_n

$$U_n = U - (x_3 + x_4)$$

$$U_n = U - x_3 - x_4$$

$$U = U_n - x_3 + x_4$$

And update U with this new value in the next 2 lower subsystem $\Sigma_3, \Sigma_2$to obtain

$$\dot{x}_2 = -x_2 - x_3(x_2^2 + x_3) + x_4 + U_2 - x_3 + x_4) \quad (45)$$

$$\dot{x}_3 = -2x_3$$

Using a Laypunov unction updated as:

$$V_{2,3}(x_3, x_2) = V_3(x_3) + 0.5x_2^2 \quad (46)$$

Analyse the subsystem such that with appropriate choice of U_n, asymptotic stability is achieved.

$$\dot{V}_{2,3}(x_3, x_2) = \dot{V}(x_3) + \frac{\partial V_2}{\partial x_2}(\dot{x}_2) < 0$$

$$\dot{x}_2 = -x_2 - x_3(x_2^2 + x_3) + x_4 + U_2 - x_3 + x_4$$

$$\dot{x}_2 = -x_2 - x_3x_2^2 - x_3^2 + 2x_4 + U_2 - x_3$$

$$\dot{x}_2 = -x_2 - x_3x_2^2 - x_3^2 + 2x_4 + U_2 - x_3$$

$$\frac{\partial V_2}{\partial x_2}(\dot{x}_2) = x_2(-x_2 - x_3x_2^2 - x_3^2 + 2x_4 + U_2 - x_3)$$

$$-x_2^2 - x_3x_2^3 - x_2x_3^2 - 2x_2x_4 + U_2x_2 - x_2x_3$$

$$\dot{V}_2(x_3, x_2) = -2x_3^2 - x_2^2 - x_3x_2^3 - x_2x_3^2 + 2x_2x_4 + x_2U_2 - x_2x_3$$

For $\dot{V} \leq -2x_3^2 - x_2^2 - x_3x_2^3 - x_2x_3^2$

$$U_2 \leq -2x_4 + x_3 \quad (47)$$

$$U_2 \leq -2x_4 + x_3$$

And update U_2 with the new value in the next lower subsystem $\Sigma_3, \Sigma_2, \Sigma_1$ to obtain

$$\dot{x}_1 = x_1 - x_2(x_2^2 + x_3^2) + x_4 + U_1 \quad (48)$$

$$U_1 = U_2 + (-x_3 + x_4)$$

$$\dot{x}_1 = x_1 - x_2(x_2^2 + x_3^2) + x_4 + U_1 + x_3 + x_4$$

$$\dot{x}_1 = -x_1 - x_2(x_2^2 + x_3^2) + x_4 - 2x_4 + x_3 + x_3 + x_4$$

$$\dot{x}_1 = -x_1 - x_2^3 - x_2x_3^2 + 2x_4 - 2x_4 + 2x_3$$

$$\dot{x}_1 = -x_1 - x_3^2 - x_2x_3^2 + 2x_3$$

Using a Laypunov function

$$\frac{\partial V}{\partial x_1} = x_1$$

$$\dot{V}(x_1) = x_1(\dot{x}_1)$$

$$= (-x_1 - x_3^2 - x_2x_3^2 + 2x_3)x_1$$

$$\dot{V}(x_1) = -x_1^2 - x_1x_3^2 - x_1x_2x_3^2 + 2x_1x_3$$

$$\dot{V}(x_1) = -x_1^2 - x_1x_3^2 - x_1x_2x_3^2 + 2x_1x_3$$

$$\dot{V}_1(x_3, x_2, x_1) = \dot{V}(x_3) + \dot{V}(x_2) + \dot{V}(x_1) < 0$$

$$= -2x_3^3 - 2x_3^2 - x_3x_2^2 - x_3x_3^2 - x_1^2 - x_1x_3^2 - x_1x_2x_3^2 + 2x_1x_3 < 0$$

$$= -2x_3^3(x_3 + 1) - x_2^2(1 + x_3) - x_1x_3^2(1 + x_2) - x_2x_3^2 - x_1^2 + 2x_1x_3 < 0$$

$$= -(2x_3^3 + x_2^2)(1 + x_3) - x_1x_3^2(1 + x_2) - x_2x_3^2 - x_1^2 + 2x_1x_3 < 0$$

$$(x_1 - 3x_3)(x_1 + x_3) = x_1^2 + x_1x_3 - 3x_3^2$$

$$(x_1 - 3x_3)(x_1 + x_3) = x_1^2 - 2x_1x_3 - 3x_3^2$$

$$-(x_1 - 3x_3)(x_1 + x_3) - 3x_3^2 = x_1^2 - 2x_1x_3$$

$$\dot{V}_1(x_3, x_2, x_1) \Rightarrow -(2x_3^3 + x_2^2)(1 + x_3) - x_1x_3^2(1 + x_2)$$

$$-x_2x_3^2 - 3x_3^2 - (x_1 - 3x_3)(x_1 + x_3) < 0$$

Therefore

$\dot{V}_1(x_3, x_2, x_1) < 0$ (Globally Asymptotically Stable)

2.4. Reformer Coupled Tanks with Disturbance

The back-stepping techniques is extended to achieve robust stabilisation of nonlinear systems in the presence of noise (disturbance) signals at each subsystem of the reformer tank using saturation functions (Akpobi, 2011). The process includes saturating the noise first and then recursively stabilising the subsystems by utilising a back-stepping technique. From (10), let:

$$z_1 = x_1$$

$z_2 = x_2 - x_1$
$\dot{Z}_1 \doteq J_1\sqrt{z_2} - J_2\sqrt{z_1}$
$\dot{Z}_2 \doteq J_2\sqrt{z_1} - 2J_1\sqrt{z_2} + K_a U$
Transform:
$M = T(z)$
$M_1 = z_1,$
$M_2 = \dot{Z}_1 = \dot{M}_1 \doteq J_1\sqrt{z_2} - J_2\sqrt{z_1}$
$\dot{M}_2 = J_1\dot{Z}_2/2\sqrt{z_2} - J_2\dot{Z}_1/2\sqrt{Z_1}$
$[J_1\dot{J}_2\sqrt{z_1} - 2J_1\sqrt{z_2}] + [K_a U/2\sqrt{z_2}] - [J_2\dot{J}_1\sqrt{z_2} - \frac{J_2\sqrt{z_1}}{2\sqrt{Z_1}}]$
Let $K_a/2\sqrt{z_2} = \varphi$ and
$[J_1\dot{J}_2\sqrt{z_1} - 2J_1\sqrt{z_2}] + [-[J_2\dot{J}_1\sqrt{z_2} - \frac{J_2\sqrt{z_1}}{2\sqrt{Z_1}}] = \tau$
Assume the disturbance of each sub system is represented$\beta_1\omega_1(m_1)$ and $\beta_2\omega_2(m_2)$

$$\left.\begin{aligned} \dot{M}_1 &= M_2 + \beta_1\omega_1(m_1) \\ \dot{M}_2 &= \tau + \beta_2\omega_2(m_2) + \varphi U \\ y &= K_s M_1 \end{aligned}\right\} \quad (49)$$

Here we consider φ a constant, let the first error be
$\epsilon_1 = y - m_1$
Taking derivative
$\dot{\epsilon}_1 = \dot{y} - \dot{m}_1 = \dot{y} - M_2 - \beta_1\omega_1(m_1)$
The Laypunov function is selected as
$V_1 = \left(\frac{1}{2}\right)\epsilon_1^{\,2}$, $\dot{V}_1 = \epsilon_1\dot{\epsilon}_1$
$\dot{V}_1 = \epsilon_1\,[\dot{y} - M_2 - \beta_1\omega_1(m_1)]$
Let the second error be $\epsilon_2 = \alpha - m_2$. Take the derivative $\dot{\epsilon}_2 = \dot{\alpha} - \dot{m}_2$ and
$\dot{\epsilon}_2 = \dot{\alpha} - \tau - \beta_2\omega_2(m_2) - \varphi U$
Then
$\dot{V}_1 = \epsilon_1\,[\dot{y} - M_2 - \beta_1\omega_1(m_1)]$
To make $\dot{V}_1 \ll 0, chose\ \alpha\ such\ that\ \alpha = \dot{y} - M_2 - \beta_1\omega_1(m_1)$, where $\beta_1\omega_1 > 0$
$\dot{V}_1 = -M_2\,\epsilon_1 - K_s M_1$
$\dot{\epsilon}_1 = -M_2 - \beta_1\omega_1(m_1)$
Taking another Laypunov function
$V_2 = V_1 + \left(\frac{1}{2}\right)\epsilon_2^{\,2}$
$\dot{V}_2 = \dot{V}_1 + \epsilon_2\dot{\epsilon}_2$
$\dot{V}_2 = \dot{V}_1 + \epsilon_2\,(\dot{\alpha} - \tau - \beta_2\omega_2(m_2) - \varphi U)$
$\dot{V}_2 = \epsilon_1\,[\dot{y} - M_2 - \beta_1\omega_1(m_1)] + \epsilon_2\,(\dot{\alpha} - \tau - \beta_2\omega_2(m_2) - \varphi U)$
Therefore the final controller input law
$U = (1/\varphi)[\epsilon_1\,[\dot{y} - M_2 - \beta_1\omega_1(m_1)] + \epsilon_2\,(\dot{\alpha} - \tau - \beta_2\omega_2(m_2) - \varphi U)\,]$

3. Results and Discussion

The performance of the reformer tank using back-stepping control has been verified by simulating it with Matlab and Simulink. From (49); for an input $u = \sin(t)$), $M_1 = 250, M_2 = 500$, $time\ (i) = 1,2,3$ and a band limited white noise as shown in Figure 2 while Figure 3 shows the system's response to inputs.

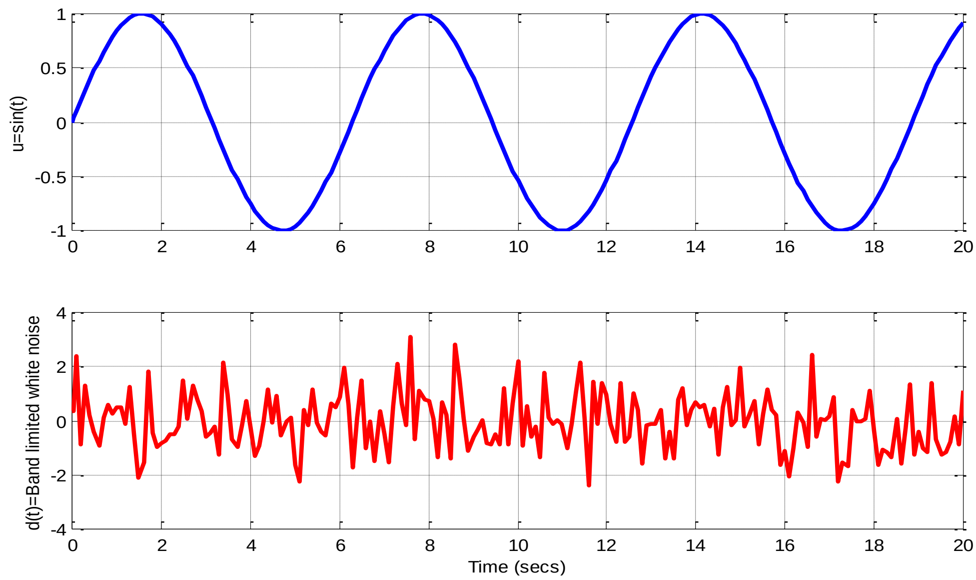

Figure 2. Input with disturbance

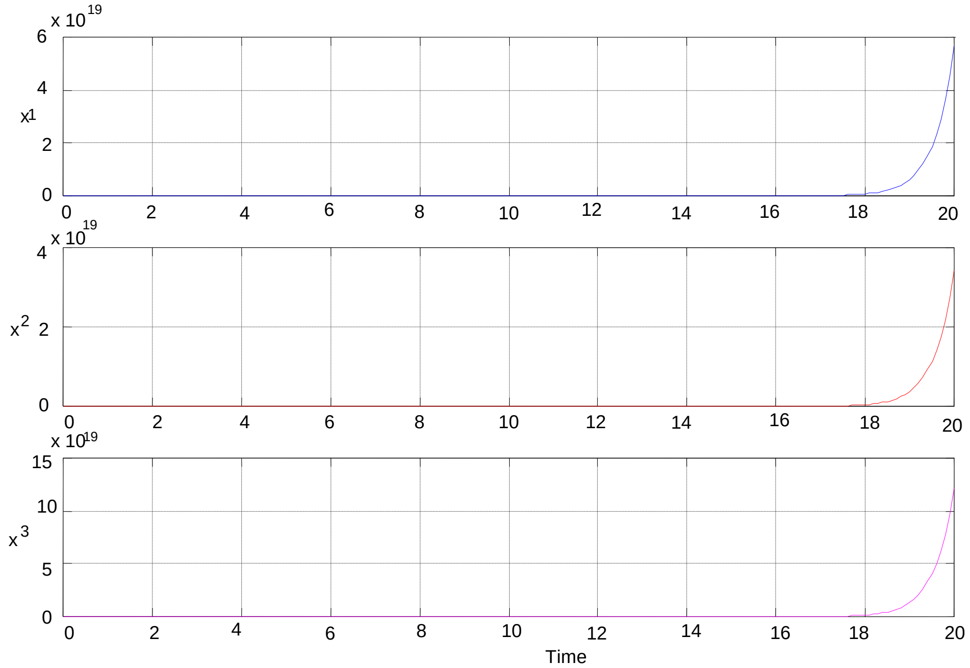

Figure 3. System response to inputs

As observed (Figure 3) the system's input (control) signal in this case is a sinusoidal wave and disturbance (white noise with limited bandwidth) are the output. The time response of the states of the original system illustrated in Figure 4, shows highly unstable motion. The deviations from the equilibrium position are so large in the order of 10^3 in 18 seconds for all the states. This is largely due to the presence of noise (disturbance).

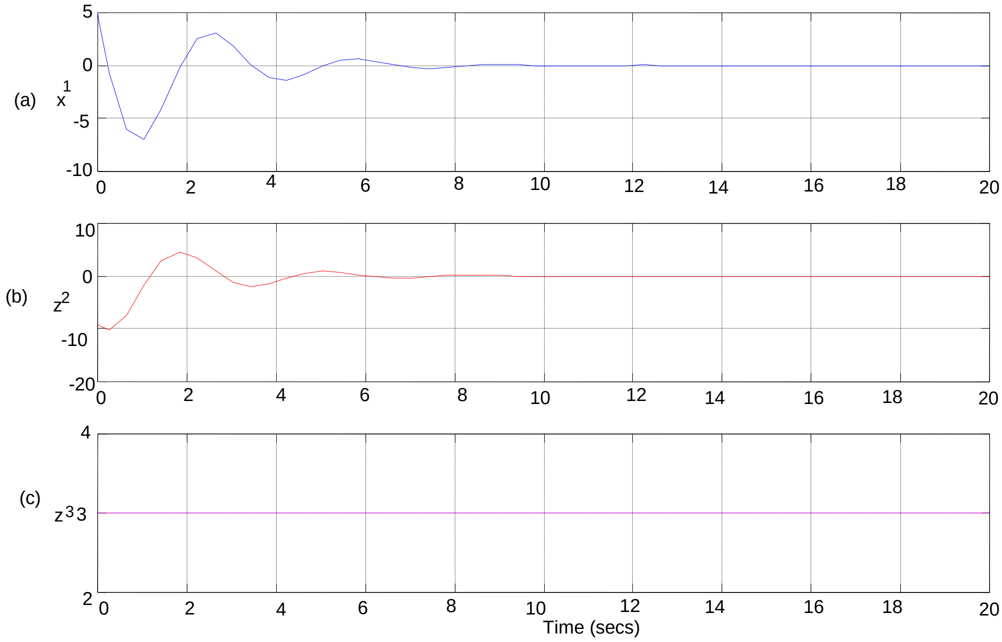

Figure 4. State trajectories of the stabilised system

In stabilising the system, the process utilised is as follows: for case: $x_1(0) = 5$, $z_2(0) = -9$, $J = 600$. For stabilised system the computer simulation of the resulting stabilised system using the designed controller derived through this procedure, shows stabilised trajectory of system's states, which were formerly noisy and unstable. The trajectory of the stabilised systems illustrates a fast response in approaching equilibrium.

4. Conclusion

The tank model comprises of uncertainty and disturbances, in other to curb this challenges feed linearization and back-stepping was used. Back-stepping controller recursively make use of Lyapunov function in each subsystem to eliminate the nonlinear terms in other to ensure proper tracking and asymptotic stability (local and global).

The feedback linearization techniques enabled the transformation of the nonlinear system into an equivalent linear system while the Kalman method was used to determine the controllability.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

Akpobi, J. A. (2011). A generalised robust stabilisation of nonlinear feedback systems in the presence of disturbance (noise) signals at each subsystem, using integrator back-stepping with saturation functions, *Proceedings of the World Congress on Engineering*, July 6 - 8, 2011, London, U.K., 576 -586.

Bacha, J., Freel, J., & Gibbs, A., (2007). *Diesel fuels technical review*, Chevron.

Bastida, H., Ponce, P., Ramirez-Mendoza, R., & Molina, A. (2013). Model and control for coupled tank using labview. *2013 IEEE International Conference on Mechatronics Electronic and Automotive Engineering*, 127- 133.

Beg, R. A., Sarker, M. R. I., & Pervez, R. (2010). Production of diesel fuel from used engine oil, *International Journal of Mechanical & Mechatronics Engineering, 10*(2): 1-6.

Boubakir, A., Boudjema, F., & Labiod, S. (2009). Euro Fuzzy sliding mode controller using nonlinear sliding surface applied to the coupled tank system. *International Journal of Automation and Computing*, *6*(1): 72-80.

John, J. A., Jaffar, N. E., & Francis, R. M. (2015). Modelling and control for coupled tank liquid level system using back-stepping Method. *International Journal of Engineering Research & Technology*, *4*(6): 667 – 671.

Kannan, S. (2012). Modelling and control of shape memory alloy actuator. *Automatic Control Engineering*. Retrieved from https://pastel.archives-ouvertes.fr/pastel-00724901 (September 8, 2018).

Khalid, H. K. (2002). *Nonlinear systems*, 3rd ed. Upper Saddle River, NJ: Prentice- Hall.

Kokotovic, P. V. (1992). The joy of feedback*:* nonlinear and adaptive. *IEEE Control Systems Magazine, 12(3): 7–17*.

Nail, B., Bekhiti, B., Bidirina, K., Kouzou, A., & Hafaifa, A. (2015). Sliding mode control and optimal GPC algorithm for coupled tanks. *3rd International Conference on Control, Engineering & Information Technology*, CEIT 2015 Tlemcen, Algeria, 25-27 May.

Naima, K., & Liazid, A. (2013). Waste oils as alternative fuel for diesel engine. *Journal of Petroleum Technology and Alternative Fuel, 4*(3): 30-43.

Ogata, K. (2002). *Modern control engineering*, Pearson Education Inc., First Indian.

Okundamiya, M. S., Emagbetere J. O., & Ogujor, E. A. (2014). Assessment of renewable energy technology and a case of sustainable energy in mobile telecommunication sector. *The Scientific World Journal 2014*, Article ID 947281, 1-13.

Slotine, J. J. E., & Li, W. (1991). *Applied nonlinear control*, Englewood cliffs, N.J: Prentice Hall.

Soltanpour, M. R., & Fateh, M. M. (2009). Sliding mode robust control of robot manipulator in the task space by support of feedback linearization and back-stepping control, *World Applied Sciences Journal, 6* (1): 70-76.

Uswarman, R., Cahyadi, A. I., & Wahyunggoro, O. (2013). Modified sliding mode control with uncertainties behaviour of a magnetic levitation system, *2013 International Conference on Robotics, Biomimetics, Intelligent Computational Systems*, 25-27 Nov., Jogjakarta, Indonesia.